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Numerical Analysis and Asymptotic Behaviour of a Family of Simultaneous Distributed-Boundary Mixed Elliptic Optimal Control Problems

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Abstract: In this paper, we consider a family of simultaneous distributed-boundary optimal control problems (P_α) on the internal energy and the heat flux for a system governed by a mixed elliptic variational equality with a parameter $\alpha > 0$ (the heat transfer coefficient on a portion of the boundary of the domain) and a simultaneous distributed-boundary optimal control problem (P) governed also by an elliptic variational equality with a Dirichlet boundary condition on the same portion of the boundary. We formulate discrete approximations $(P_{h\alpha})$ and (P_h) of the optimal control problems (P_α) and (P) respectively, for each $h > 0$ and for each $\alpha > 0$, through the finite element method with Lagrange's triangles of type 1 with parameter h (the longest side of the triangles). The goal of this paper is to study the convergence of this family of discrete simultaneous distributed-boundary mixed elliptic optimal control problems $(P_{h\alpha})$ when the parameters α goes to infinity and the parameter h goes to zero simultaneously. We prove the convergence of the family of discrete problems $(P_{h\alpha})$ to the discrete problem (P_h) when $\alpha \to +\infty$, for each $h > 0$, in adequate functional spaces. We study the convergence of the discrete problems $(P_{h\alpha})$ and (P_h), for each $\alpha > 0$, when $h \to 0^+$ obtaining a commutative diagram which relates the continuous and discrete simultaneous distributed-boundary mixed elliptic optimal control problems $(P_{h\alpha}), (P_\alpha), (P_h)$ and (P) by taking the limits $h \to 0^+$ and $\alpha \to +\infty$ respectively. We also study the double convergence of $(P_{h\alpha})$ to (P) when $(h, \alpha) \to (0^+, +\infty)$ which represents the diagonal convergence in the above commutative diagram.

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