Nonsmooth Problems with Applications in Mechanics
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# Regularity and Numerical Analysis for Incompressible Miscible Flows with non-Smooth Diffusion-Dispersion Tensor in Porous Media

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**Abstract:** Applications of incompressible miscible flows in porous media can be found in various engineering areas, such as reservoir simulations and exploration of underground water and oil. In last several decades, numerous efforts have been made in developing efficient numerical methods and in establishing rigorous numerical analysis. In most existing works, one often assumes that diffusion-dispersion tensor involved is smooth enough, while it is not always true in practice. In this talk, we focuses on an incompressible miscible flow in porous media with the commonly-used Bear–Scheidegger diffusion-dispersion tensor $D(\mathbf{u}) = \Phi d_m I + |\mathbf{u}|\big(\alpha_T I + (\alpha_L - \alpha_T)\frac{\mathbf{u}\otimes\mathbf{u}}{|\mathbf{u}|^2}\big)$, which does not satisfy the regularity condition even for a smooth velocity field $\mathbf{u}$. Therefore, analyses in those previous works on optimal $L^\infty((0,T);L^2)$ error estimates may not be valid for Galerkin-Galerkin methods, Galerkin-mixed methods and many other numerical methods. A new approach is presented recently, in terms of a parabolic projection, which only requires the Lipschitz continuity of $D(\mathbf{u})$. With the new approach, we establish optimal $L^p$ error estimates and an almost optimal $L^\infty$ error estimate.