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Weak Solutions of the Hall-Magnetohydrodynamics Equations on $\mathbb{R}^3$

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Abstract: Magnetohydrodynamics describes the motion of electrically conductive fluid in the presence of a magnetic field with wide range of applications in geophysics and astrophysics. We consider the following Hall-MHD system on $[0,T]\times\mathbb{R}^3$

$$
\begin{aligned}
&d\mathbf{u}+\Big[(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p-s\,(\mathbf{B}\cdot\nabla)\mathbf{B}+s\,\nabla\Big(\frac{|\mathbf{B}|^2}{2}\Big)-\nu_1\,\Delta\mathbf{u}\Big]\,dt \;=\; \mathbf{f}_1(t),\\
&d\mathbf{B}+\Big[(\mathbf{u}\cdot\nabla)\mathbf{B}-(\mathbf{B}\cdot\nabla)\mathbf{u}+\varepsilon\,\mathrm{curl}[(\mathrm{curl}\mathbf{B})\times\mathbf{B}]-\nu_2\,\Delta\mathbf{B}\Big]\,dt \;=\; \mathbf{f}_2(t),\\
&\mathrm{div}\,\mathbf{u} \;=\; 0 \quad \text{and} \quad \mathrm{div}\mathbf{B} \;=\; 0
\end{aligned}
$$

supplemented by the initial conditions $\mathbf{u}(0)=\mathbf{u}_0$ and $\mathbf{B}(0)=\mathbf{u}_0$. In this problem $\mathbf{u}(t,x)$, $\mathbf{B}(t,x)$ for $(t,x)\in[0,T]\times\mathbb{R}^3$, are three-dimensional vector fields representing velocity and magnetic field, respectively, and the real valued function $p(t,x)$ denotes the pressure of the fluid. The positive constants ν_1,ν_2,s represent kinematic viscosity, resistivity and the Hartmann number, respectively. Using the Fourier analysis and the compactness method we prove the existence of a weak solution.