Nonsmooth Problems with Applications in Mechanics
Bedlewo, Poland, June 17-22, 2023

Elastoplasticity with Softening in Spring Network Models: a Dual Approach Via a State-Dependent Sweeping Process

Ivan Gudoshnikov

Institute of Mathematics of the Czech Academy of Sciences
Prague, Czech Republic

gudoshnikov@math.cas.cz

Abstract: Softening plasticity and fracture mechanics lead to ill-posed mathematical problems due to the loss of monotonicity. Multiple co-existing solutions are possible when softening elements are coupled together, and solutions cannot be continued beyond the point of complete failure of a material. Moreover, spatially continuous models with softening suffer from localization of strains and stresses to measure-zero submanifolds.

We formulate a problem of quasistatic evolution of elasto-plastic spring networks (Lattice Spring Models) with a plastic flow rule which describes linear hardening, linear softening and perfectly plastic springs in a uniform manner. The fundamental kinematic and static characteristics of the network are described by the rigidity theory and structural mechanics. To solve the evolution problem we convert it to a type of a differential quasi-variational inequality known as the state-dependent sweeping process. We prove the existence of solution to the associated time-stepping problem (implicit catch-up algorithm), and the estimates we obtain imply the existence of a solution to the (time-continuous) sweeping process.

Using numerical simulations of regular grid-shaped networks with softening we demonstrate the development of non-symmetric shear bands. At the same time, in toy examples it is easy to analytically derive multiple co-existing solutions, appearing in a bifurcation which happens when the parameters of the networks continuously change from hardening through perfect plasticity to softening.