Nonsmooth Problems with Applications in Mechanics
Bedlewo, Poland, June 17-22, 2023

# On the Optimal Control of Moreau's Sweeping Process

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**Abstract:** Moreau's sweeping process with a drift $f$ is the evolution inclusion

$$\dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \quad \text{a.e.,} \tag{D}$$

where $C(t)$ is a moving closed set, with external normal cone $N_{C(t)}(\cdot)$, $f$ is Lipschitz and $u$ is a control. The above problem together with the initial condition $x(0) = x_0 \in C(0)$ admits a unique forward in time solution, provided the sets $C(t)$ satisfy a regularity condition called uniform prox-regularity. This condition generalizes both convexity and $\mathcal{C}^{1,1}$-regularity of the boundary of $C(t)$. The dynamics (D) can be seen as a constrained evolution, where the constraint is active in it.

The optimal control of (D) provides significant challenges, as its right hand side severely lacks Lipschitz regularity with respect to the state variable $x$. I will present the state of the art on necessary optimality conditions, that are of the type of Pontryagin Maximum Principle. The main available approaches rely either on discretization or on different types of penalizations, and are due to a number of different authors, including Mordukhovich and collaborators, Hermosilla and Palladino, De Pinho, Ferreira and Smirnov, Zeidan and collaborators, and myself with collaborators.