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Stabilization of a Beam Equation with Time Delay

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Abstract: We consider the following beam equation with delay:

$$(E)\quad \begin{cases} \frac{\partial^2 z(x,t)}{\partial t^2} = -\frac{\partial^4 z(x,t)}{\partial x^4} + v(t)\frac{\partial z(x,t-r)}{\partial t}, & (x,t) \in (0,1)\times(0,+\infty) \\ z(\sigma,t) = \frac{\partial^2 z(\sigma,t)}{\partial x^2} = 0, & \sigma = 0,1,\ t\in(0,+\infty) \\ z(x,t) = \psi(x,t), & (x,t)\in[0,+\infty)\times[-r,0] \end{cases}$$

where $\psi \in C(\mathbb{R}_*^+ \times [-r,0], \mathbb{R})$, $r = \frac{2}{\pi}$ and the scalar-valued function $v \in L^2(\mathbb{R}^+)$ is the control. The problem (E), represents a particular model of the theory of elasticity describing how solid objects deform and become stressed out due to prescribed loading conditions. In the literature, both the strong and partial stabilization have been studied for the problem (E) without delay $(r = 0)$.

This work aims to add a marginal improvement to the feedback stabilization question of the beam equation with a discrete delay. The strong stabilization has been established via an explicit delayed bounded control. The necessity of inserting a time lag in the classical model $(r = 0)$, might increase the understanding of how external loads impact the solid. For example, time delay can be related to : slow rotation, retardation of lateral displacement, etc. Therefore, it partially enhances the precision of the model. In compromise, the existence of delay may cause some impractical instabilities in the system. Hence, a stabilization study is required.